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Charged Kerr–NUT Metric with Lambda-Term and Step-by-Step Extension Method

Xu Dian-Yan[1] and Qiu Zong-Yan[2]

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The charged Kerr–NUT metric with Λ term is derived using a step-by-step extension method.

1. INTRODUCTION

More than 30 years ago, Newman *et al.* (1963) discovered a new metric, now called the NUT metric. The NUT metric is one of the exact solutions of the Einstein vacuum field equations and is different from the Schwarzschild metric.

A number of papers related to the NUT variable l have since been published (Demianski and Newman, 1966; Demianski, 1973; Kinnersley, 1969; Frolov, 1974; Misner, 1963; Quevedo and Mashhoom, 1991). The most complicated treatments are Frolov (1974) and Quevedo and Mashhoom (1991).

Frolov dealt with the Kerr–NUT–de Sitter metric in the retarded time coordinate by using the Newman–Penrose (NP) spin formalism. His main result is his metric equation (6.9), which reduces to the Kerr–de Sitter's metric when the NUT variable $l = 0$.

The disadvantages of Frolov's approach are as follows:

1. The constants involved in equation (6.9) are ambiguous and obscure. Perhaps this originates from the direct solution of the NP spin equations. If we substitute the metric (6.9) back into the Einstein field equations in de Sitter space in metric form.

[1] Department of Computer Science and Technology, Peking University, Beijing 100871, China.
[2] Department of Information Sciences, School of Mathematical Sciences, Peking University, Beijing 100871, China.

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$$R_{\alpha\beta} + \Lambda g_{\alpha\beta} = 0$$

some of the constants may be "fixed" in form, i.e., expressed by physical parameters, such as m, a, l, and Λ.

2. When $\Lambda = 0$, Frolov's metric (6.9) cannot be reduced to the Kerr–NUT metric of Kinnersley (1969); in particular, $g_{u\phi}$ is incorrect. From the dimensional point of view, $g_{u\phi}$ is obviously incorrect. $g_{u\phi}$ involves five terms. The dimension of $mr(C_2 - \rho^{02})$ is 4, while the dimension of each of the remaining terms is 2.

3. The Kerr–de Sitter metric given by Frolov is irregular in the neighborhood of the rotation axis.

One of the disadvantages of Queredo and Mashhoom's work is that they did not give the explicit form of the charged Kerr–NUT metric in quasi-Boyer–Lindquist coordinates.

The main aims of the present paper are to derive the charged Kerr–NUT metric in de Sitter space and to demonstrate our step-by-step extension (SSE) method.

2. THE STEP-BY-STEP EXTENSION METHOD

2.1 The NUT Metric

The line element of the NUT metric is (Newman *et al.*, 1963)

$$\begin{aligned} ds^2 = {} & \left[-\frac{2(mr + l^2)}{r^2 + l^2} \right] dt^2 \\ & - \frac{1}{[1 - 2(mr + l^2)/(r^2 + l^2)]} dr^2 - (r^2 + l^2)\, d\theta^2 \\ & - 4l \cos\theta \left[1 - \frac{2(mr + l^2)}{r^2 + l^2} \right] dt\, d\phi - \left\{ (r^2 + l^2) \sin^2\theta \right. \\ & \left. - 4\, l^2 \cos^2\theta \left[1 - \frac{2(mr + l^2)}{r^2 + l^2} \right] \right\} d\phi^2 \end{aligned} \tag{1}$$

It is easy to show that the metric (1) is singular or violates the local flatness condition in the neighborhood of the symmetry axis (Kramer *et al.*, 1980). This slight disadvantage can be remedied or nonsingularized by changing $(-\cos\theta)$ to $(1 - \cos\theta)$ for $0 \le \theta \le \pi/2$, and to $(1 + \cos\theta)$ for $\pi/2 \le \theta \le \pi$, in equation (1). The line element becomes

$$ds^2 = \left[1 - \frac{2(mr + l^2)}{r^2 + l^2}\right] dt^2 - \frac{1}{[1 - 2(mr + l^2)/(r^2 + l^2)]} dr^2$$
$$+ 4l(1 \mp \cos\theta)\left[1 - \frac{2(mr + l^2)}{r^2 + l^2}\right] dt d\phi - (r^2 + l^2)\, d\theta^2$$
$$- \left\{(r^2 + l^2)\sin^2\theta - 4l^2(1 \mp \cos\theta)^2\left[1 - \frac{2(mr + l^2)}{r^2 + l^2}\right]\right\} d\phi^2 \qquad (2)$$

Obviously, the metric (2) is local or exhibits elementary flatness.

2.2 First Step of the Extension

In this subsection we will extend the NUT metric to include the Λ term. The suggested metric is as follows:

$$ds^2 = \left[1 - \frac{2(mr + l^2)}{r^2 + l^2} + B(r)\right] dt^2 - \frac{1}{[1 - 2(mr + l^2)/(r^2 + l^2) + B(r)]} dr^2$$
$$- (r^2 + l^2)\, d\theta^2 + 4l(1 \mp \cos\theta)\left[1 - \frac{2(mr + l^2)}{r^2 + l^2} + B(r)\right] dt\, d\phi$$
$$- \left\{(r^2 + l^2)\sin^2\theta\right.$$
$$\left. - 4l^2(1 \mp \cos\theta)^2\left[1 - \frac{2(mr + l^2)}{r^2 + l^2} + B(r)\right]\right\} d\phi^2 \qquad (3)$$

where $B(r)$ is an unknown function to be determined.

The Einstein–Maxwell equations with a Λ term are given by

$$R_{\alpha\beta} - \tfrac{1}{2} g_{\alpha\beta}R - \Lambda g_{\alpha\beta} = -2[F_\alpha{}^\lambda F_{\beta\lambda} - \tfrac{1}{4} g_{\alpha\beta}F_{\lambda\tau}F^{\lambda\tau}] \qquad (4a)$$

(4a) can be recast as

$$R_{\alpha\beta} + \Lambda g_{\alpha\beta} = -2[F_\alpha{}^\lambda F_{\beta\lambda} - \tfrac{1}{4} g_{\alpha\beta}F_{\lambda\tau}F^{\lambda\tau}] \qquad (4b)$$

When the electromagnetic (EM) field $F_{\alpha\beta} = 0$, (4b) reduces to

$$R_{\alpha\beta} + \Lambda g_{\alpha\beta} = 0 \qquad (5)$$

Now consider the (22)-component of equation (5), i.e.,

$$R_{22} + \Lambda g_{22} = 0 \qquad (6)$$

From equation (3) we can evaluate R_{22}, which is

$$R_{22} = B(r) - [B'(r)]r - \frac{2r^2}{r^2 + l^2} B(r) \tag{7}$$

Combining equations (6) and (7), we get

$$B(r) - [B'(r)]r - \frac{2r^2}{r^2 + l^2} B(r) - \Lambda(r^2 + l^2) = 0 \tag{8}$$

The solution of equation (8) is

$$B(r) = -\frac{1}{3}\Lambda \frac{1}{r^2 + l^2}[r^4 + 6l^2r^2 - 3l^4] \tag{9}$$

One can check that the solution satisfies all the remaining components of equation (5).

2.3 Second Step of the Extension

Now we take the second step in the extension of the NUT metric. The suggested metric is

$$\begin{aligned} ds^2 = {} & \left[1 - \frac{2(mr + l^2)}{r^2 + l^2} + D(r)\right] dt^2 - \frac{1}{[1 - 2(mr + l^2)/(r^2 + l^2) + D(r)]} dr^2 \\ & + 4l(1 \mp \cos\theta)\left[1 - \frac{2(mr + l^2)}{r^2 + l^2} + D(r)\right] dt\, d\phi - (r^2 + l^2)\, d\theta^2 \\ & - \Bigg\{(r^2 + l^2)\sin^2\theta \\ & - 4l^2(1 \mp \cos\theta)^2\left[1 - \frac{2(mr + l^2)}{r^2 + l^2} + D(r)\right]\Bigg\} d\phi^2 \end{aligned} \tag{10}$$

where

$$\begin{aligned} D(r) &= B(r) + H(r) \\ B(r) &= -\frac{1}{3}\Lambda \frac{r^4 + 6r^2l^2 - 3l^4}{r^2 + l^2} \end{aligned} \tag{11}$$

$H(r)$ is another unknown function to be determined.

The related or associated EM potential is

$$A_\alpha\, dx^\alpha = \frac{Qr}{r^2 + l^2}[dt - 2l(1 \mp \cos\theta)\, d\phi] \tag{12}$$

Similar to Section 2.2, we can compute R_{22} from equation (10); the final result is

$$R_{22} = D(r) - [D'(r)]r - \frac{2r^2}{r^2 + l^2} D(r)$$

$$= B(r) + H(r) - [B'(r) + H'(r)]r - \frac{2r^2}{r^2 + l^2} [B(r) + H(r)] \quad (13)$$

Upon utilization of equation (13), it is easy to show that

$$R_{22} + \Lambda g_{22} = H(r) - [H'(r)]r - \frac{2r^2}{r^2 + l^2} H(r) \quad (14)$$

From equation (12) we get

$$T_{22} = -2[F_2{}^{\lambda} F_{2\lambda} - \tfrac{1}{4} g_{22} F_{\lambda\tau} F^{\lambda\tau}] \quad (15)$$

$$= \frac{Q^2}{r^2 + l^2}$$

The (22)-component of the Einstein–Maxwell equations with Λ term

$$R_{22} + \Lambda g_{22} = T_{22} \quad (16)$$

becomes

$$H(r) - [H'(r)]r - \frac{2r^2}{r^2 + l^2} H(r) = \frac{Q^2}{r^2 + l^2} \quad (17)$$

The solution of (17) is

$$H(r) = \frac{Q^2}{r^2 + l^2} \quad (18)$$

The charged NUT metric in de Sitter space is

$$ds^2 = \left[1 - \frac{2(mr + l^2)}{r^2 + l^2} + \frac{Q^2}{r^2 + l^2} - \frac{1}{3} \Lambda \frac{r^4 + 6r^2 l^2 - 3l^4}{r^2 + l^2} \right] dt^2$$
$$- \frac{1}{\left[1 - \frac{2(mr + l^2)}{r^2 + l^2} + \frac{Q^2}{r^2 + l^2} - \frac{1}{3} \Lambda \frac{r^4 + 6r^2 l^2 - 3l^4}{r^2 + l^2} \right]} dr^2$$
$$- (r^2 + l^2)\, d\theta^2$$
$$+ 4l(1 \mp \cos\theta) \left[1 - \frac{2(mr + l^2)}{r^2 + l^2} + \frac{Q^2}{r^2 + l^2} \right.$$
$$\left. - \frac{1}{3} \Lambda \frac{r^4 + 6r^2 l^2 - 3l^4}{r^2 + l^2} \right] dt\, d\phi$$

$$-\left\{(r^2+l^2)\sin^2\theta - 4l^2(1\mp\cos\theta)^2\left[1-\frac{2(mr+l^2)}{r^2+l^2}\right.\right.$$
$$\left.\left.+\frac{Q^2}{r^2+l^2}-\frac{1}{3}\Lambda\frac{r^4+6r^2l^2-3l^4}{r^2+l^2}\right]\right\}d\phi^2 \tag{19}$$

3. KERR–NUT–DE SITTER METRIC

In this section, we derive the Kerr–NUT–de Sitter metric from the Kerr–NUT metric (Kinnersley, 1969) by the SSE method. The suggested metric is

$$ds^2=\frac{1}{\Sigma_l}[\Delta_\lambda-\Delta_\sigma a^2\sin^2\theta]\,dt^2-\frac{\Sigma_l}{\Delta_\lambda}dr^2-\frac{\Sigma_l}{\Delta_\sigma}d\theta^2+\frac{2N}{\Sigma_l}dt\,d\phi$$
$$-\frac{1}{\Sigma_l}\{\Delta_\sigma(r^2+a^2+l^2)^2\sin^2\theta-\Delta_\lambda(a\sin^2\theta-2l\cos\theta)^2\}\,d\phi^2 \tag{20}$$

where

$$\begin{aligned}
\Sigma_l &= r^2+(l+a\cos\theta)^2\\
\Delta_l &= r^2-2mr+a^2\cos^2\theta-l^2\\
&= \Delta-a^2\sin^2\theta\\
\Delta &= r^2-2mr-l^2+a^2\\
\Delta_\lambda &= r^2+a^2-2mr-l^2+B(r,\theta)\\
&= \Delta+B(r,\theta)\\
\Delta_\sigma &= 1+S(r,\theta)\\
N &= [\Delta_\sigma(r^2+a^2+l^2)-\Delta_\lambda]a\sin^2\theta+(2l\cos\theta)\,\Delta_\lambda
\end{aligned} \tag{21}$$

Here $B(r,\theta)$, $S(r,\theta)$ are two unknown functions to be determined. In what follows we choose that $B(r,\theta)=B(r)$, $S(r,\theta)=S(\theta)$. Note that each of the unknown functions $B(r,\theta)$, $S(r,\theta)$ is proportional to the cosmological constant Λ. When $\Lambda=0$, we have $B(r)=0$, $S(\theta)=0$, and the metric (20) reduces to the Kerr–NUT metric in quasi-Boyer–Lindquist coordinates.

In the calculation, we frequently use the following important identity:

$$\begin{aligned}
&\Sigma_l^2\,\Delta_\lambda\Delta_\sigma\sin^2\theta-N^2\\
&\quad=[\Delta_\lambda-\Delta_\sigma a^2\sin^2\theta][\Delta_\sigma(r^2+a^2\\
&\qquad+l^2)^2\sin^2\theta-\Delta_\lambda(a\sin^2\theta-2l\cos\theta)^2]
\end{aligned} \tag{22}$$

From the suggested metric (20), one can evaluate the Christoffel symbols of the first and second kinds $[\alpha\beta, \lambda]$, $\Gamma^{\lambda}{}_{\alpha\beta}$, and the Ricci tensor $R_{\alpha\beta}$; for example,

$$\begin{aligned} R_{00} &= R^{\alpha}{}_{0\alpha 0} \\ &= \partial_{\alpha}\Gamma^{\alpha}{}_{00} - \partial_{0}\Gamma^{\alpha}{}_{\alpha} + \Gamma^{\alpha}{}_{\lambda\alpha}\Gamma^{\lambda}{}_{00} - \Gamma^{\alpha}{}_{\lambda 0}\Gamma^{\lambda}{}_{0\alpha} \end{aligned} \tag{23}$$

After lengthy and tedious but straightforward calculation, we arrive at the important immediate stage expression of R_{00},

$$R_{00} = R^{(0)}_{00} + R^{(1)}_{00} + R^{(2)}_{00} \tag{24}$$

where

$$R^{(0)}_{00} = 0 \tag{25}$$

$$\begin{aligned} R^{(1)}_{00} &= \frac{1}{\Sigma_l^3} B(\Delta + a^2\sin^2\theta) - \frac{1}{\Sigma_l^3}(\Delta + a^2\sin^2\theta) r\frac{\partial B}{\partial r} + \frac{1}{2\Sigma_l^2}\Delta\frac{\partial^2 B}{\partial r^2} \\ &+ \frac{1}{\Sigma_l^3} S[2a^2\sin^2\theta(-a^2 + mr + l^2) - 2(la\cos\theta + a^2\cos^2\theta)\Delta] \\ &- \frac{1}{\Sigma_l^3} a\sin\theta(l + a\cos\theta)(\Delta + a^2\sin^2\theta)\frac{\partial S}{\partial\theta} \\ &- \frac{3}{2\Sigma_l^2} a^2\sin\theta\cos\theta\frac{\partial S}{\partial\theta} - \frac{1}{2\Sigma_l^2} a^2\sin^2\theta\frac{\partial^2 S}{\partial\theta^2} \end{aligned} \tag{26}$$

$$\begin{aligned} R^{(2)}_{00} &= \frac{1}{\Sigma_l^3}[B^2 - S^2a^4\sin^4\theta] - \frac{1}{\Sigma_l^3}[B + Sa^2\sin^2\theta] r\frac{\partial B}{\partial r} + \frac{1}{2\Sigma_l^2} B\frac{\partial^2 B}{\partial r^2} \\ &- \frac{1}{\Sigma_l^3}(l + a\cos\theta)\left[a\sin\theta\frac{\partial S}{\partial S} + 2Sa\cos\theta\right][B + Sa^2\sin^2\theta] \\ &- \frac{1}{2\Sigma_l^2} a^2\sin^2\theta\, S\frac{\partial^2 S}{\partial\theta^2} + \frac{1}{\Sigma_l^2} a^2\sin^2\theta\, S^2 \\ &- \frac{3}{2\Sigma_l^2} a^2\sin\theta\cos\theta\, S\frac{\partial S}{\partial\theta} \end{aligned} \tag{27}$$

Mathematically, since the cosmological constant Λ can take arbitrary values, at least in certain intervals or ranges, the field equations can be divided into

$$R^{(1)}_{00} + \Lambda g^{(0)}_{00} = 0 \tag{28a}$$

$$R^{(2)}_{00} + \Lambda g^{(1)}_{00} = 0 \tag{28b}$$

where

$$g^{(0)}_{00} = \frac{1}{\Sigma_l}[\Delta - a^2\sin^2\theta] \tag{29a}$$

$$g^{(1)}_{00} = \frac{1}{\Sigma_l}[B(r) - S(\theta)a^2\sin^2\theta] \tag{29b}$$

Through calculation we find that equation (28a) can be further decomposed into two other independent equations, only one of which involves m. From these three independent field equations, finally we find

$$B(r) = -\tfrac{1}{3}\Lambda[r^4 + a^2r^2 + 6l^2r^2 - 3l^4 + 3a^2l^2] \tag{30}$$

$$S(\theta) = \tfrac{1}{3}\Lambda(4l + a\cos\theta)a\cos\theta \tag{31}$$

Take the coordinate transformation

$$t \to t' = \sqrt{A}t \qquad \phi \to \phi' = \sqrt{A}\phi \tag{32}$$

or

$$dt \to dt' = \sqrt{A}\,dt \qquad d\phi \to d\phi' = \sqrt{A}\,d\phi \tag{33}$$

where A is a positive constant, independent of coordinates t, r, θ, ϕ. The metric (20) becomes

$$\begin{aligned} ds^2 = {} & A\frac{1}{\Sigma_l}[\Delta_\lambda - \Delta_\sigma a^2\sin^2\theta]\,dt^2 - \frac{\Sigma_l}{\Delta_\lambda}dr^2 - \frac{\Sigma_l}{\Delta_\sigma}d\theta^2 + \frac{2A}{\Sigma_l}N\,dt\,d\phi \\ & - A\frac{1}{\Sigma_l}\{\Delta_\sigma(r^2 + a^2 + l^2)^2 - \Delta_\lambda[a\sin^2\theta - 2l\cos\theta]^2\}\,d\phi^2 \\ \Delta_\lambda = {} & r^2 + a^2 - 2mr - l^2 + B(r) = \Delta + B(r) \\ \Delta_\sigma = {} & 1 + S(\theta) \\ B(r) = {} & -\tfrac{1}{3}\Lambda[r^4 + a^2r^2 + 6l^2r^2 - 3l^4 + 3a^2l^2] \\ S(\theta) = {} & \tfrac{1}{3}\Lambda(4l + a\sin\theta)a\cos\theta \end{aligned} \tag{34}$$

By performing the coordinate transformation

$$\begin{aligned} dt \to dt' &= du + \frac{1}{\sqrt{A\Delta_\lambda}}(r^2 + a^2 + l^2)dr \\ d\phi \to d\phi' &= d\phi + \frac{1}{\sqrt{A\Delta_\lambda}}a\,dr \end{aligned} \tag{35}$$

where u is the retarded time, we can transform the metric (34) into

$$ds^2 = A\frac{1}{\Sigma_l}[\Delta_\lambda - \Delta_\sigma a^2\sin^2\theta]\,du^2 + 2\sqrt{A}\,du\,dr + 2A\frac{N}{\Sigma_l}du\,d\phi$$
$$- 2\sqrt{A}(a\sin^2\theta - 2l\cos\theta)\,dr\,d\phi - \frac{\Sigma_l}{\Delta_\sigma}d\theta^2$$
$$- A\frac{1}{\Sigma_l}\{\Delta_\sigma(r^2 + a^2 + l^2)^2\sin^2\theta$$
$$- \Delta_\lambda[a\sin^2\theta - 2l\cos\theta]^2\}\,d\phi^2 \tag{36}$$

The solution or metric (20), (21), (30), (31) satisfies the other components of the Einstein vacuum field equations with Λ term. In addition, when $\Lambda = 0$, and we set $A = 1$, our solution or metric (36) directly reduces to the Kerr–NUT metric (3.47) of Kinnersley (1969). Finally, the Kerr–de Sitter metric given by Frolov (1974) does not have local flatness in the vicinity of the rotation axis. This can be remedied by introducing the factor A and putting

$$A = [1 + \tfrac{1}{3}\Lambda a^2]^{-2} \tag{37}$$

4. CHARGED KERR–NUT METRIC IN DE SITTER SPACE

We will derive the charged Kerr–NUT–de Sitter metric from the charged Kerr–NUT metric and the Kerr–NUT–de Sitter metric by the SSE method. The suggested metric and EM potential are

$$ds^2 = \frac{1}{\Sigma_l}[\Delta_\lambda - \Delta_\sigma a^2\sin^2\theta]\,dt^2 - \frac{\Sigma_l}{\Delta_\lambda}dr^2 - \frac{\Sigma_l}{\Delta_\sigma}d\theta^2 + 2\frac{N}{\Sigma_l}dt\,d\phi$$
$$- \frac{1}{\Sigma_l}[\Delta_\sigma(r^2 + a^2 + l^2)^2\sin^2\theta$$
$$- \Delta_\lambda(a\sin^2\theta - 2l\cos\theta)^2]d\phi^2 \tag{38}$$

$$A_\alpha dx^\alpha = \frac{Qr}{\Sigma_l}[dt - (a\sin^2\theta - 2l\cos\theta)d\phi] \tag{39}$$

where

$$\Delta_l = r^2 - 2mr + Q^2 - l^2 + a^2\cos^2\theta = \Delta - a^2\sin^2\theta$$
$$\Delta = r^2 - 2mr + Q^2 - l^2 + a^2$$
$$\Delta_\lambda = r^2 + a^2 - 2mr + Q^2 - l^2 + B(r) = \Delta + B(r) \tag{40}$$
$$\Delta_\sigma = 1 + S(\theta)$$

$$N = \Delta_\sigma(r^2 + a^2 + l^2)a\sin^2\theta - \Delta_\lambda(a\sin^2\theta - 2l\cos\theta)$$
$$\Sigma_l = r^2 + (l + a\cos\theta)^2$$

$B(r)$ and $S(\theta)$ are unknown functions to be determined.

From equations (38)–(40) we can derive

$$\begin{aligned} R_{00} = {} & \frac{1}{\Sigma_l^3}[\Delta_\lambda^2 - \Delta_\sigma^2 a^4\sin^4\theta] - \frac{r}{\Sigma_l^3}[\Delta_\lambda - \Delta_\sigma a^2\sin^2\theta]\frac{\partial\Delta_\lambda}{\partial r} + \frac{1}{2\Sigma_l^2}\Delta_\lambda\frac{\partial^2\Delta_\lambda}{\partial r^2} \\ & - \frac{1}{\Sigma_l^3}[\Delta_\lambda + \Delta_\sigma a^2\sin^2\theta](l + a\cos\theta)\left[a\sin\theta\frac{\partial\Delta_\sigma}{\partial\theta} + 2\Delta_\sigma a\sin\theta\right] \\ & + \frac{1}{\Sigma_l^2}\Delta_\sigma^2 a^2\sin^2\theta - \frac{3}{2\Sigma_l^2}\Delta_\sigma a^2\sin\theta\cos\theta\frac{\partial\Delta_\sigma}{\partial\theta} \\ & - \frac{1}{2\Sigma_l^2}\Delta_\sigma a^2\sin^2\theta\frac{\partial^2\Delta_\sigma}{\partial\theta^2} \end{aligned} \tag{41}$$

etc., and

$$\begin{aligned} T_{00} &= -2[F_0{}^\lambda F_{0\lambda} - \tfrac{1}{4}g_{00}F_{\lambda\tau}F^{\lambda\tau}] \\ &= \frac{Q^2}{\Sigma_l^3}[\Delta_\lambda + \Delta_\sigma a^2\sin^2\theta] \end{aligned} \tag{42}$$

etc.

Next, upon utilization of the decomposition relations

$$\Delta_\lambda = \Delta + B(r), \qquad \Delta_\sigma = 1 + S(\theta) \tag{43}$$

and using procedures similar to those of section 3, we arrive at

$$B(r) = -\tfrac{1}{3}\Lambda[r^4 + a^2r^2 + 6r^2l^2 - 3l^4 + 3a^2l^2] \tag{44}$$
$$S(\theta) = \tfrac{1}{3}\Lambda[4l + a\sin\theta]a\cos\theta \tag{45}$$

Equations (38), (44), and (45) are the same as equations (20), (30), and (31), except that Δ, and Δ of (38) involve an additional term Q^2. The functions $B(r)$, $S(\theta)$ are independent of the EM field, or Q^2, and are, to the same extent, similar to the $l = 0$ case.

5. DISCUSSION

When $l = 0$, the metric (38) and the EM potential reduce to the Kerr–Newman–de Sitter metric and the related EM potential.

When $\Lambda = 0$, the metric (38) reduces to the charged Kerr–NUT metric.

When $Q = 0$, the metric (38) reduces to the Kerr–NUT–de Sitter metric.

The metric (38) together with (39), (40) are checked by directly substituting them into the Einstein–Maxwell equations with Λ term for other components.

This work can be extended to cases which involve the gravitational multipole moment.

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